

2. ASTROPHYSICAL CONSTANTS AND PARAMETERS

Table 2.1. Revised February 2012 by E. Bergren and D.E. Groom (LBNL). The figures in parentheses after some values give the $1-\sigma$ uncertainties in the last digit(s). Physical constants are from Ref. 1. While every effort has been made to obtain the most accurate current values of the listed quantities, the table does not represent a critical review or adjustment of the constants, and is not intended as a primary reference.

The values and uncertainties for the cosmological parameters depend on the exact data sets, priors, and basis parameters used in the fit. Many of the derived parameters reported in this table have non-Gaussian likelihoods. Parameters may be highly correlated, so care must be taken in propagating errors. Unless otherwise specified, cosmological parameters are from six-parameter fits to a flat Λ CDM cosmology using 7-year WMAP data alone [2]. For more information see Ref. 3 and the original papers.

Quantity	Symbol, equation	Value	Reference, footnote
speed of light	c	$299\,792\,458 \text{ m s}^{-1}$	exact[4]
Newtonian gravitational constant	G_N	$6.673\,8(8) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	[1]
Planck mass	$\sqrt{\hbar c/G_N}$	$1.220\,93(7) \times 10^{19} \text{ GeV}/c^2$ $= 2.176\,51(13) \times 10^{-8} \text{ kg}$	[1]
Planck length	$\sqrt{\hbar G_N/c^3}$	$1.616\,20(10) \times 10^{-35} \text{ m}$	[1]
standard gravitational acceleration	g_N	$9.806\,65 \text{ m s}^{-2} \approx \pi^2$	exact[1]
jansky (flux density)	Jy	$10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$	definition
tropical year (equinox to equinox) (2011)	yr	$31\,556\,925.2 \text{ s} \approx \pi \times 10^7 \text{ s}$	[5]
sidereal year (fixed star to fixed star) (2011)		$31\,558\,149.8 \text{ s} \approx \pi \times 10^7 \text{ s}$	[5]
mean sidereal day (2011) (time between vernal equinox transits)		$23^h\,56^m\,04^s 090\,53$	[5]
astronomical unit	au, A	$149\,597\,870\,700(3) \text{ m}$	[6]
parsec (1 $au/1$ arc sec)	pc	$3.085\,677\,6 \times 10^{16} \text{ m} = 3.262 \dots \text{ly}$	[7]
light year (deprecated unit)	ly	$0.306\,6 \dots \text{ pc} = 0.946\,053 \dots \times 10^{16} \text{ m}$	
Schwarzschild radius of the Sun	$2G_NM_\odot/c^2$	$2.953\,250\,077\,0(2) \text{ km}$	[8]
Solar mass	M_\odot	$1.988\,5(2) \times 10^{30} \text{ kg}$	[9]
Solar equatorial radius	R_\odot	$6.9551(4) \times 10^8 \text{ m}$	[10]
Solar luminosity	L_\odot	$3.828 \times 10^{26} \text{ W}$	[11]
Schwarzschild radius of the Earth	$2G_NM_\oplus/c^2$	$8.870\,055\,94(2) \text{ mm}$	[12]
Earth mass	M_\oplus	$5.972\,6(7) \times 10^{24} \text{ kg}$	[13]
Earth mean equatorial radius	R_\oplus	$6.378\,137 \times 10^6 \text{ m}$	[5]
luminosity conversion (deprecated)	L	$3.02 \times 10^{28} \times 10^{-0.4} M_{\text{bol}} \text{ W}$	[14]
flux conversion (deprecated)	\mathcal{F}	(M_{bol} = absolute bolometric magnitude = bolometric magnitude at 10 pc) $2.52 \times 10^{-8} \times 10^{-0.4} m_{\text{bol}} \text{ W m}^{-2}$ (m_{bol} = apparent bolometric magnitude))	from above
ABsolute monochromatic magnitude	AB	$-2.5 \log_{10} f_\nu - 56.10$ (for f_ν in $\text{W m}^{-2} \text{ Hz}^{-1}$) $= -2.5 \log_{10} f_\nu + 8.90$ (for f_ν in Jy)	[15]
Solar circular velocity v_0 at R_0 from Galactic center	v_0/R_0	$30.2 \pm 0.2 \text{ km s}^{-1} \text{ kpc}^{-1}$	[16]
Solar distance from Galactic center	R_0	$8.4(4) \text{ kpc}$	[17]
circular velocity at R_0	v_0 or Θ_0	$240(10) \text{ km s}^{-1}$	[18]
local disk density	ρ_{disk}	$3-12 \times 10^{-24} \text{ g cm}^{-3} \approx 2-7 \text{ GeV}/c^2 \text{ cm}^{-3}$	[19]
local dark matter density	ρ_χ	canonical value $0.3 \text{ GeV}/c^2 \text{ cm}^{-3}$ within factor 2–3	[20]
escape velocity from Galaxy	v_{esc}	$498 \text{ km/s} < v_{\text{esc}} < 608 \text{ km/s}$	[21]
present day CMB temperature	T_0	$2.7255(6) \text{ K}$	[22]
present day CMB dipole amplitude		$3.355(8) \text{ mK}$	[2]
Solar velocity with respect to CMB		$369(1) \text{ km/s towards } (\ell, b) = (263.99(14)^\circ, 48.26(3)^\circ)$	[2]
Local Group velocity with respect to CMB	v_{LG}	$627(22) \text{ km/s towards } (\ell, b) = (276(3)^\circ, 30(3)^\circ)$	[23]
entropy density/Boltzmann constant	s/k	$2\,889.2 (T/2.725)^3 \text{ cm}^{-3}$	[14]
number density of CMB photons	n_γ	$410.5 (T/2.725)^3 \text{ cm}^{-3}$	[24]
baryon-to-photon ratio	$\eta = n_b/n_\gamma$	$6.19(15) \times 10^{-10}$ $5.1 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}$ (95% CL)	[25]
number density of baryons	n_b	$(2.54 \pm 0.06) \times 10^{-7} \text{ cm}^{-3}$ $(2.1 \times 10^{-7} < n_b < 2.7 \times 10^{-7}) \text{ cm}^{-3}$ (95% CL)	from η in [2]
present day Hubble expansion rate	H_0	$100 h \text{ km s}^{-1} \text{ Mpc}^{-1} = h \times (9.777\,752 \text{ Gyr})^{-1}$	from η in [25]
scale factor for Hubble expansion rate	h	$0.710(25) \text{ WMAP7; WMAP7}\oplus\text{Cepheids}=0.721(17)$	[2,27]
Hubble length	c/H_0	$0.925\,063 \times 10^{26} h^{-1} \text{ m} = 1.28(5) \times 10^{26} \text{ m}$	
scale factor for cosmological constant	$c^2/3H_0^2$	$2.852 \times 10^{51} h^{-2} \text{ m}^2 = 5.5(5) \times 10^{51} \text{ m}^2$	
critical density of the Universe	$\rho_c = 3H_0^2/8\pi G_N$	$2.775\,366\,27 \times 10^{11} h^2 M_\odot \text{ Mpc}^{-3}$ $= 1.878\,47(23) \times 10^{-29} h^2 \text{ g cm}^{-3}$ $= 1.053\,75(13) \times 10^{-5} h^2 (\text{GeV}/c^2) \text{ cm}^{-3}$	
baryon density of the Universe	$\Omega_b = \rho_b/\rho_c$	$\dagger 0.0226(6) h^{-2} = \dagger 0.045(3)$	[2,3]
cold dark matter density of the universe	$\Omega_{\text{cdm}} = \rho_{\text{cdm}}/\rho_c$	$\dagger 0.111(6) h^{-2} = \dagger 0.22(3)$	[2,3]
dark energy density of the Λ CDM Universe	Ω_Λ	$\dagger 0.73(3)$	[2,3]
pressureless matter density of the Universe	$\Omega_m = \Omega_{\text{cdm}} + \Omega_b$	0.27 ± 0.03 (From Ω_Λ and flatness constraint)	[2,3]
dark energy equation of state parameter	w	$\dagger -0.98 \pm 0.05$ (WMAP7+BAO+ H_0)	[28]
CMB radiation density of the Universe	$\Omega_\gamma = \rho_\gamma/\rho_c$	$2.471 \times 10^{-5} (T/2.725)^4 h^{-2} = 4.75(23) \times 10^{-5}$	[24]
neutrino density of the Universe	Ω_ν	$0.0005 < \Omega_\nu h^2 < 0.025 \Rightarrow 0.0009 < \Omega_\nu < 0.048$	[29]
total energy density of the Universe (curvature)	$\Omega_{\text{tot}} = \Omega_m + \dots + \Omega_\Lambda$	$\dagger 1.002 \pm 0.011$ (WMAP7+BAO+ H_0)	[2,3]

2 2. Astrophysical constants

Quantity	Symbol, equation	Value	Reference, footnote
fluctuation amplitude at $8 h^{-1}$ Mpc scale	σ_8	$\dagger 0.80(3)$	[2,3]
curvature fluct. amplitude at $k_0 = 0.002 \text{ Mpc}^{-1}$	$\Delta_{\mathcal{R}}^2$	$\dagger 2.43(11) \times 10^{-9}$	[2,3]
scalar spectral index	n_s	$\dagger 0.963(14)$	[2,3]
running spectral index slope, $k_0 = 0.002 \text{ Mpc}^{-1}$	$dn_s/d\ln k$	$\dagger -0.03(3)$	[2]
tensor-to-scalar field perturbations ratio, $k_0 = 0.002 \text{ Mpc}^{-1}$	$r = T/S$	$\dagger < 0.36$ at 95% CL	[2,3]
redshift at decoupling	z_{dec}	$\dagger 1091(1)$	[2]
age at decoupling	t_*	$\dagger 3.79(5) \times 10^5 \text{ yr}$	[2]
sound horizon at decoupling	$r_s(z_*)$	$\dagger 147(2) \text{ Mpc}$	[2]
redshift of matter-radiation equality	z_{eq}	$\dagger 3200 \pm 130$	[2]
redshift of reionization	z_{reion}	$\dagger 10.5 \pm 1.2$	[2]
age at reionization	t_{reion}	$430_{-70}^{+90} \text{ Myr}$	[2,30]
reionization optical depth	τ	$\dagger 0.088(15)$	[2,3]
age of the Universe	t_0	$\dagger 13.75 \pm 0.13 \text{ Gyr}$	[2]

\ddagger Parameter in six-parameter Λ CDM fit [2].

\dagger Derived parameter in six-parameter Λ CDM fit [2].

\ddag Extended model parameter [2].

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D. Larson *et al.*, *Astrophys. J. Supp.* **192**, 16 (2011);
E. Komatsu *et al.*, *Astrophys. J. Supp.* **192**, 18 (2011).
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- B.W. Petley, *Nature* **303**, 373 (1983).
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- While A is approximately equal to the semi-major axis of the Earth’s orbit, it is not exactly so. Nor is it exactly the mean Earth-Sun distance. There are a number of reasons: a) the Earth’s orbit is not exactly Keplerian due to relativity and to perturbations from other planets; b) the adopted value for the Gaussian gravitational constant k is not exactly equal to the Earth’s mean motion; and c) the mean distance in a Keplerian orbit is not equal to the semi-major axis a : $\langle r \rangle = a(1 + e^2/2)$, where e is the eccentricity. (Discussion courtesy of Myles Standish, JPL).
- The distance at which 1 A subtends 1 arc sec: 1 A divided by $\pi/648\,000$.
- Product of $2/c^2$ and the heliocentric gravitational constant $G_N M_\odot = A^3 k^2 / 86400^2$, where k is the Gaussian gravitational constant, $0.017\,202\,098\,95$ (exact) [5]. The value and error for A given in this table are used.
- Obtained from the $G_N M_\odot$ product [5] and G_N [1].
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- $4\pi A^2 \times (1361 \text{ W m}^{-2})$ [31]. Assumes isotropic irradiance.
- Schwarzschild radius of the Sun (above) scaled by the Earth/Sun mass ratio given in Ref. 5.
- Obtained from the $G_N M_\oplus$ product [5] and G_N [1].
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The IAU (Commission 36) has recommended $3.055 \times 10^{28} \text{ W}$ for the zero point. Based on newer Solar measurements, the value and significance given in the table seems more appropriate.
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- D. Scott & G.F. Smoot, “Cosmic Microwave Background,” in this *Review*.
- $n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT}{\hbar c} \right)^3$ and $\rho_\gamma = \frac{\pi^2}{15} \frac{(kT)^4}{(\hbar c)^3 c^2} ; \frac{kT_0}{\hbar c} = 11.900(4)/\text{cm}$.
- B.D. Fields, S. Sarkar, “Big-Bang Nucleosynthesis,” this *Review*.
- Conversion using length of sidereal year.
- Average of WMAP7 [2] and independent Cepheid-based measurement by A.G. Riess *et al.*, *Astrophys. J.* **730**, 119 (2011). Other high-quality measurements could have been included.
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- $\Omega_\nu h^2 = \sum m_{\nu_j} / 93 \text{ eV}$, where the sum is over all neutrino mass eigenstates. The lower limit follows from neutrino mixing results reported in this *Review* combined with the assumptions that there are three light neutrinos ($m_\nu < 45 \text{ GeV}/c^2$) and that the lightest neutrino is substantially less massive than the others: $\Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2$, so $\sum m_{\nu_j} \geq m_{\nu_3} \approx \sqrt{\Delta m_{32}^2} = 0.05 \text{ eV}$. (This becomes 0.10 eV if the mass hierarchy is inverted, with $m_{\nu_1} \approx m_{\nu_2} \gg m_{\nu_3}$.) Astrophysical determinations of $\sum m_{\nu_j}$, reported in the Full Listings of this *Review* under “Sum of the neutrino masses,” range from $< 0.17 \text{ eV}$ to $< 2.3 \text{ eV}$ in papers published since 2003. Alternatively, if the limit obtained from tritium decay experiments ($m_\nu < 2 \text{ eV}$) is used for the upper limit, then $\Omega_\nu < 0.04$.
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